

# A $k_T$ -dependent sea-quark density for CASCADE

Martin Hentschinski



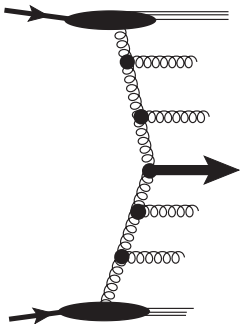
IFT Autonoma University  
Madrid/CSIC

Based on results obtained with F. Hautmann and H. Jung

# Outline

- 1 Motivation
- 2 Definition of unintegrated density
- 3 Numerical analysis
- 4 Conclusions

# Introduction



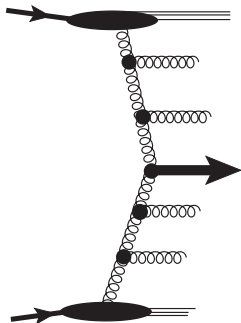
CASCADE: Monte-Carlo event generator based on the CCFM evolution equation

- designed for dynamics at small  $x$
- unintegrated gluon density  $\mathcal{A}(x, k_t, \mu^2)$   
+ CCFM parametrization of valence quark distribution
- but no sea quark distribution/density

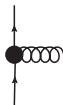
theoretical basis:  $k_T$ -factorization at small  $x$  [Catani, Hautmann '94]

- resummation of collinear (DGLAP) and small  $x$  (BFKL) logarithms can be achieved at a time in a consistent way
- CASCADE: MonteCarlo realization of  $k_T$ -factorization at small  $x$
- based on CCFM: LO evolution equation which interpolates between DGLAP and BFKL

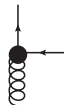
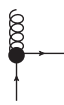
# CCFM evolution and quark emission



CCFM evolution based on principle of color coherence  
 → emissions of **gauge bosons**



unintegrated gluon and  
valence quark



not present

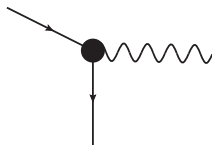
**Consequences:** (A) Evolution (exclusive radiative corrections!):

- only gluonic emissions, no quark → jets purely gluonic
- DGLAP: naturally contained
- BFKL: through NLO corrections, not contained in (LO) CCFM evolution

# Quark splitting: hard processes

**Consequences:** (B) hard process: LO (sea-)quark induced processes require 1-loop ME (and higher)

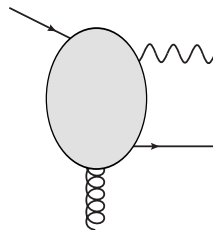
- EXAMPLE: DY/Z-boson production



- DGLAP @ leading order:  $q\bar{q} \rightarrow Z$
- quark  $q$ : valence quark of proton 1
- anti-quark  $\bar{q}$ : sea quark, couples to gluon evolution of proton 2

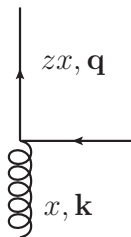
CCFM with unintegrated gluon:

- Forward DY (sea & valence quark)':  $qg^* \rightarrow Zq$   
 $\mathcal{O}(\alpha_s)$  [Ball, Marzani, '09]
- Central DY (2 seaquarks):  $g^*g^* \rightarrow Zq\bar{q}$   $\mathcal{O}(\alpha_s^2)$   
[Deak, Schwennsen, '08], [Baranov, Lipatov, Zotov '08]
- Collinear divergence: require finite quark masses and/or cut-offs



# Goal of this study: gluon $\rightarrow$ quark splitting ( $P_{qg}$ )

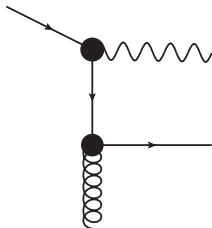
- supplement CCFM evolution by gluon  $\rightarrow$  quark splitting
  - restrict to splitting in the last evolution step
  - keep finite transverse quark momentum  $q_T$   
 $\rightarrow k_T$  factorized seaquark
  - correct high energy & collinear limits,  
 $\rightarrow$  similar to CCFM evolution
- + test accuracy of (formal) factorization numerically



Process of interest at LHC: **forward Drell-Yan** production ( $\gamma^*, Z, W$ )

- probe proton at very small  $x$ , up to  $3 \cdot 10^{-6}$
- investigate small  $x$  dynamics: BFKL, saturation, ...
- allows to compare exact versus factorized expression

# Quark-gluon splitting and collinear factorization



- **DGLAP:** contains naturally splitting function

$$P_{qg}(z) = \text{Tr}(z^2 + (1-z)^2)$$

- no  $k_T$  dependence for seaquark distribution  $q(x, \mu^2)$  and partonic cross-section  $\sigma_{q\bar{q} \rightarrow Z}$
- no small  $x$  dynamics included

$$\hat{\sigma}_{q\bar{q} \rightarrow Z}(\nu = \hat{s}) = \underbrace{\sqrt{2}G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2)$$

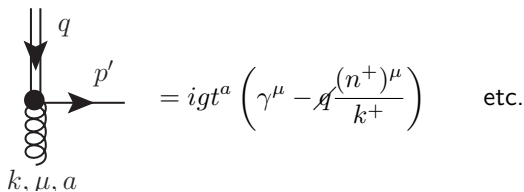
[Catani, Hautmann '94] : high energy resummation within collinear factorization:  **$k_T$ -dependent splitting function**

$$P_{qg}^{\text{CH}}(z, \mathbf{k}^2, \mathbf{q}^2) = T_R \left( \frac{q^2}{q^2 + z(1-z)\mathbf{k}^2} \right)^2 \left[ P_{qg}(z) + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{q^2} \right]$$

- $\otimes$  gluon Green's function: high energy resummed splitting
- universal  $\rightarrow$  defines small  $x$ -resummed seaquark distribution
- full  $k_T$  (gluon) dependence, but integrate out  $q_T$  (quark)

# gauge invariant off-shell factorization: reggeized quarks

- **reggeized quarks** (in analogy to reggeized gluons for BFKL):
  - at high energies, effective d.o.f. in  $t$ -channel processes with quark exchange [Fadin, Sherman, 76,77 ], [Lipatov, Viazovsky, '00], [Bogdan, Fadin, 06],
  - here applied to  $qg^* \rightarrow Zq$  process at Born level
- **effective vertices**: re-arrangement of QCD diagrams



$$=igt^a \left( \gamma^\mu - \not{A} \frac{(n^+)^{\mu}}{k^+} \right) \quad \text{etc.}$$

→ gauge invariant definition of off-shell Matrix Elements

$$\hat{\sigma}_{q\bar{q}^* \rightarrow Z}(\nu, \mathbf{q}^2) = \underbrace{\sqrt{2}G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2 - \mathbf{q}^2)$$

- gluon-quark splitting =  $T_R$ : Multi-Regge-Kinematics sets  $z = 0$

# $k_T$ -factorized seaquark

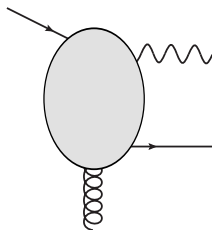
- limit  $z \rightarrow 0$  only asymptotically justified
- goal: keep  $z$  finite  $\rightarrow$  correct & complete collinear limit
- how ?  $\rightarrow$  generalize effective vertices
- here: possible to include  $z \neq 0$  + keeping off-shell gauge invariance  
 $\rightarrow$  re-do calculation: re-obtain  $k_T$ -dependent splitting function  
 by Catani&Hautmann

$$\mathcal{A}^{\text{sea}}(x, q^2, \mu^2) := \frac{1}{q^2} \int_x^1 dz \int_0^{\mu^2/z} d\mathbf{k}^2 P_{qg}^{\text{CH}}(z, \mathbf{k}^2, q^2) \mathcal{A}_{\text{CCFM}}^{\text{gluon}}\left(\frac{x}{z}, \mathbf{k}^2, \bar{\mu}^2\right)$$

$q_T$ -dependent sea-quark density:

- \* correct collinear limit & small  $x$  resummation ( CH-splitting + gluon density) & gauge invariance verified
- \* two choices for the hard scale  $\bar{\mu}^2$ : factorization scale  $\bar{\mu}^2 = \mu^2$  (inclusive) or angular ordering scale  $\bar{\mu}^2 = \frac{q^2 + (1-z)\mathbf{k}^2}{(1-z)^2}$  (CCFM)

# Forward DY: exact versus factorized



confront with full  $\hat{\sigma}_{qg^* \rightarrow Zq}$  (in  $k_T$ -fact.) [Ball, Marzani, '09]:  
define 'renormalized' cross-section  $\bar{\sigma}_{qg^* \rightarrow Zq}$

$$\bar{\sigma}(\nu, \mathbf{k}^2) \equiv \hat{\sigma}(\nu, \mathbf{k}^2) - \int_x^1 \frac{dz}{z} \int \frac{d\mathbf{q}^2}{\mathbf{q}^2} \hat{\sigma}_{q\bar{q}^* \rightarrow Z} P_{qg}^{\text{CH}}$$

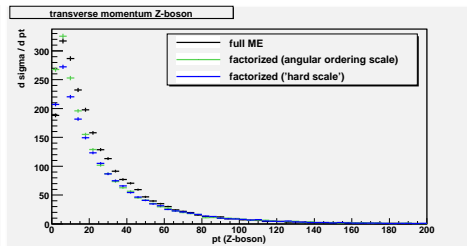
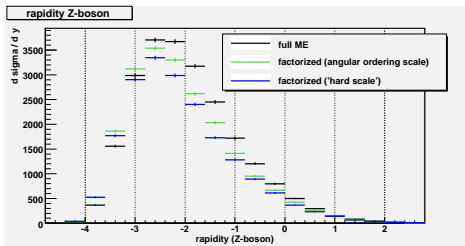
- 'renormalized'  $\bar{\sigma}$  subleading in high energy ( $\hat{s}_{qg^*} \gg Q^2, \mathbf{q}^2, \mathbf{k}^2$ ) and collinear ( $Q^2 \gg \mathbf{q}^2 \gg \mathbf{k}^2$ ) limit  $\rightarrow$  'higher order' correction
- factorized expression has approximate kinematics ( $\hat{\sigma}_{q\bar{q}^* \rightarrow Z}$ )

$$\delta(z\nu - M_Z^2 - \mathbf{q}^2) \leftrightarrow \delta(z\nu - M_Z^2 - \frac{\mathbf{q}^2}{1-z} - z\mathbf{k}^2)$$

- $k_T$ -factorization increases accuracy in kinematics, but does not capture finite  $z$ -correction

# Numerical comparison full ME versus factorized

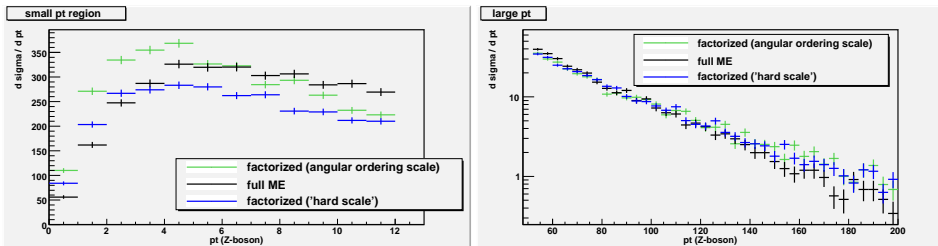
- numerical value of  $\sigma_{\text{tot}}$  of factorized expression smaller than full ME
- reason:  $s$ -channel contributions, mainly kinematics



- plots: scale of running coupling for factorized/full ME:  
 $\alpha_s(Q^2)$  with  $Q^2 = p_Z^2 + M_Z^2$

# Numerical comparison full ME versus factorized

Agreement best for large  $p_T$  region



'Renormalized'  $q\bar{q}^* \rightarrow Zq$  cross-section

$$\bar{\sigma}(\nu, k^2) \equiv \hat{\sigma}(\nu, k^2) - \int_x^1 \frac{dz}{z} \int \frac{d\mathbf{q}^2}{\mathbf{q}^2} \hat{\sigma}_{q\bar{q}^* \rightarrow Z} P_{qg}^{\text{CH}}$$

yields finite (7% – 16%) correction to factorized expression, free of large collinear logarithms

# Conclusion and outlook

- Defined  $q_T$  dependent seaquark density
  - (●) interpolates (as CCFM) between DGLAP and high energy limit
  - (●) gauge invariant definition of off-shell splitting and ME
  - (●) 'renormalized' 1-loop cross-section  $\bar{\sigma}_{qg^* \rightarrow Zq}$  collinear finite
- Numerical checks:
  - (●) Qualitative agreement of exact and factorized expression
  - (●) Approximation in kinematics  $\rightarrow$  factorized ME generally below complete calculation
  - (●)  $\bar{\sigma}_{qg^* \rightarrow Zq}$  gives finite correction to leading order (i.e.  $q_T$ -factorized) expression
- CASCADE: Splitting allows to include gluon-quark splitting into CCFM evolution
  - (●) starting point to systematically include quark emissions into parton shower
  - (●) seaquark induced processes on the same level as gluon induced processes

# Scales, masses, couplings, parton densities

Scales:

- Z-mass:  $M_Z = 91.1876 \text{ GeV}$

Unintegrated gluon density:

- CCFM set A0

Valence quark distribution:

- CCFM parametrization valence quark distribution
  - starting distribution CTEQ
  - evolution with  $P_{qq}$  and angular ordering of emitted gluon

Coupling constants:

- $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$
- $\alpha_s(Q^2)$  with  $Q^2 = M_Z^2 + p_Z^2$

## Z-mass distribution

